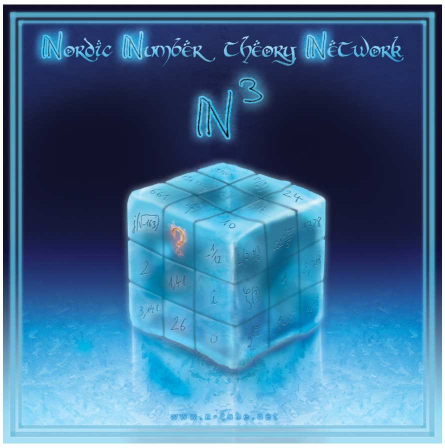
Nordic Number theory Network Days XVI

Copenhagen University,
May 13 – May 14, 2022
organised by Nuno Hultberg (Copenhagen) and Fabien Pazuki (Copenhagen).



Program

	Friday 13.05			Saturday 14.05
13:00-13:10	*Foreword*	◇		
13:10-14:00	**Cécile Armana**	◇	09:10-10:00	**Mahya Mehrabdollahei**
14:00-14:20	*Coffee break*	◇	10:00-10:20	*Coffee break*
14:20-15:10	**Jori Merikoski**	◇	10:20-11:10	**Didier Lesesvre**
15:10-15:30	*Coffee break*	◇	11:10-11:30	*Coffee break*
15:30-16:20	**Andreas Mihatsch**	◇	11:30-12:20	**Jerson Caro**
16:20-16:40	*Coffee break*	◇	12:30	*Farewell*
16:40-17:30	**Michael Björklund**	◇		
18:30	*Social event*			

Abstracts

TIME: Friday 13, 13:10-14:00.
ROOM: Aud 10.
SPEAKER: **Cécile Armana** (Besançon, France).
TITLE: *Sturm bounds for Drinfeld-type automorphic forms over function fields.*
ABSTRACT: Sturm bounds say how many successive Fourier coefficients are sufficient to determine a modular form of a given weight and level. For classical modular forms, they also provide explicit bounds for the number of Hecke operators generating the Hecke algebra. I will review the situation over the rational function field $\mathbb{F}_q(t)$ for "Drinfeld-type" automorphic forms and their Hecke algebra. Sturm bounds are obtained using refinements of a fundamental domain for a Bruhat-Tits tree under the action of a congruence subgroup. This is a joint work with Fu-Tsun Wei (National Tsing Hua University, Taiwan).

TIME: Friday 13, 14:20-15:10.
ROOM: Aud 10.
SPEAKER: **Jori Merikoski** (Oxford, UK).
TITLE: *The polynomials $X^2 + (Y^2 + 1)^2$ and $X^2 + (Y^3 + Z^3)^2$ also capture their primes.*
ABSTRACT: We show that there are infinitely many primes of the form $X^2 + (Y^2 + 1)^2$ and $X^2 + (Y^3 + Z^3)^2$. Our work builds on the famous Friedlander-Iwaniec result on primes of the form $X^2 + Y^4$. More precisely, Friedlander and Iwaniec obtained an asymptotic formula for the number of primes of this form. For the argument we need to estimate Type II sums, which is achieved by an application of the Weil bound, both for point-counting and for exponential sums over curves. The type II information we get is too narrow for an asymptotic formula, but we can apply Harman's sieve method to establish a lower bound of the correct order of magnitude for the number of primes of the form $X^2 + (Y^2 + 1)^2$ and $X^2 + (Y^3 + Z^3)^2$.

TIME: Friday 13, 15:30-16:20.
ROOM: Aud 10.
SPEAKER: **Andreas Mihatsch** (Bonn, Germany).
TITLE: *On the linear Arithmetic Fundamental Lemma.*
ABSTRACT: The linear Arithmetic Fundamental Lemma (AFL) is a conjectural identity between certain intersection numbers on deformation spaces of p-divisible groups and derivatives of orbital integrals. It is a local building block for a conjectural global identity of intersection numbers of cycles on Shimura varieties and derivatives of L-functions. In this talk, I will provide an introduction to the AFL identity and present an argument for a reduction from the non-basic to the basic case. This is joint work with Qirui Li.

Time: Friday 13, 16:40-17:30.
Room: Aud 10.
Speaker: **Michael Björklund** (Göteborg, Sweden).
Title: *Probabilistic Limit Laws in Multiplicative Diophantine Approximation.*
Abstract: In this talk I will summarize some recent results of myself, Reynold Fregoli (UZH) and Alexander Gorodnik (UZH), which are concerned with the (Lebesgue almost sure) asymptotic behavior of the number of solutions to some well-studied inequalities in multiplicative Diophantine approximation. The techniques are mostly dynamical in nature, and exploit quantitative equidistribution of expanding translates of certain unipotent orbits in spaces of lattices. If time permits I will also discuss other probabilistic limit laws (like CLTs and extreme values) for this type of counting which can be attained with similar methods.

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Time: Saturday 14, 09:10-10:00.
Room: Aud 10.
Speaker: **Mahya Mehrabdollahei** (Paris, France).
Title: *Limit of the Mahler measures.*
Abstract: Mahler measure is a type of height function for univariate and multivariate polynomials P, defined as the arithmetic mean of $\log|P|$ over the torus. The study of sequences of Mahler measures was first started by Boyd and Lawton with the hope to answer Lehmer's Conjecture. Though the conjecture is still open, Boyd-Lawton Theorem asserts that for every multivariable polynomial P there exists a sequence of univariable polynomials P_n such that the Mahler measure of P is the limit of the Mahler measures of P_n. In this talk, we will see a generalization of this theorem where the polynomials P_n are also multivariate. Moreover, we will see an application of this generalization for determining the limit of a sequence of the Mahler measure of specific bivariate polynomials, called P_d. This is a joint work with Brunault, Guilloux and Pengo.

Time: Saturday 14, 10:20-11:10.
Room: Aud 10.
Speaker: **Didier Lesesvre** (Lille, France).
Title: *The Weyl law with uniform power savings.*
Abstract: For a compact Riemannian manifold, the Weyl law describes the asymptotic behavior of the number of eigenvalues of the underlying Laplace operator. Understanding lower order or error terms remains particularly challenging. In the more general context of locally symmetric spaces, the spectral theory of the Laplacian is intimately related to the theory of automorphic forms (among which are elliptic curves, modular or Maass forms, Galois representations...) and similar questions arise.

It is therefore natural to ask for such a Weyl law to hold for families of all automorphic forms of a given reductive group. Until recently, however, all the known asymptotics were for automorphic forms with fixed aspects. In some sense, this amounts to picking a "slice" of the space of automorphic forms only. Unfortunately, making explicit the hidden dependencies in the featured error term does not allow to sum over these aspects to obtain a uniform counting law: existing results did not allow to patch back together the slices.

In their recent achievement, Brumley and Milicevic obtained a uniform Weyl law for GL(2), using the trace formula of Arthur, but with essentially no error term. Simplifying the very general setting of this work, and going back to ideas used a long time ago by Drinfeld in the setting of function fields, we obtained a power savings in the Weyl law for the universal family of all automorphic forms of GL(2). The idea is to study a suitable "conductor zeta function", and to deduce a counting law by Tauberian arguments.

TIME: Saturday 14, 11:10-12:20.
ROOM: Aud 10.
SPEAKER: **Jerson Caro** (Santiago, Chile).
TITLE: *Explicit quadratic Chabauty over number fields.*
ABSTRACT: In 1922 Mordell conjectured that every nonsingular curve C defined over $\mathbb{Q}$ with $g(C) > 1$ has only finitely many rational points. Chabauty in 1941 achieved a partial proof of that conjecture, the case $\mathrm{rank}_{\mathbb{Z}} J(\mathbb{Q}) < g(C)$ where J is the Jacobian of C. Chabauty proved that for a prime of good reduction for C, the set $C(\mathbb{Q}_p) \cap \overline{J(\mathbb{Q})}$ is finite, where $\overline{J(\mathbb{Q})}$ denotes the p-adic closure of $J(\mathbb{Q})$ in $J(\mathbb{Q}_p)$. In particular, the number of rational points of C is finite. It was until 1983 that Faltings showed the conjecture without restrictions in the Jacobian of the curve C. Meanwhile, Coleman using the same ideas as Chabauty, with the same hypothesis, gave the following upper bound for the number of rational points of C,

$$\#C(\mathbb{Q}) \leq \#C(\mathbb{F}_p) + (2g(C) - 2),$$

whenever $p > 2g(X)$. The aim of this talk is to show a new process that generalize the previous result for hyperbolic surfaces inside abelian varieties of Mordell-Weil rank 1, under some geometric condictions over a special fiber of X. Our process is based on overdetermined ω-integrality in positive characteristic. This is joint work with my PhD. thesis advisor Hector Pasten.