

Nordic Number Theory Network Days XIV

University of Uppsala May 28 – May 29, 2021 organised by Anders Karlsson (UU & Geneva), Matthew Palmer (UU) and Andreas Strömbergsson (UU)

Program

All times are CEST (UTC+2).

28th May	
14:00-14:50	Emmanuel Breuillard
15:00-15:50	Alexander Gorodnik
16:00-16:20	Coffee break
16:20-17:10	Sary Drappeau
17:20-18:10	Nirvana Coppola

29th May	
09:00-09:50	Paul Ziegler
10:00-10:50	Asbjørn Nordentoft
11:00-11:20	Coffee break
11:20-12:10	Julia Brandes
12:20-13:10	Pavel Kurasov

Abstracts

SPEAKER: Emmanuel Breuillard (University of Cambridge)

TITLE: Irreducibility of random polynomials with large degree

ABSTRACT: I will discuss properties of random polynomials with integer coefficients and large degree. Modulo GRH we have shown with Peter Varju that they are irreducible modulo cyclotomic factors, thus offering a conditional answer to a question of Odlyzko and Poonen. The proof is based on an understanding of mixing rates of random walks on the affine line over large finite fields and has connections with the Lehmer conjecture. If time permits I will discuss further applications of this method.

SPEAKER: Alexander Gorodnik (University of Zurich)

TITLE: Random Diophantine Geometry

ABSTRACT: We explore a series of counting and density problems involving "generic" lattices and explain how ergodic-theoretic methods can be used to study them.

SPEAKER: Sary Drappeau (Institut de Mathématiques de Marseille)

TITLE: Continued fractions and limit laws for values of L-functions

ABSTRACT: A theorem due to Selberg shows that the values $\zeta(1/2 + it)$, t at random in [0, T], tend to distribute according to a complex Gaussian of variance $1/2 \log \log T$ as T grows. Analogous statements are expected to hold for other families of L functions having Euler products.

Let d(n) be the number of divisors of n. The values referred to in the title are the central values of the Estermann function, defined for rational x by

$$D(1/2, x) = \sum_{n \ge 1} d(n) n^{-1/2} e^{2\pi i n x},$$

understood as the value at s = 1/2 of the analytic continuation of the same sum with n^{-s} instead of $n^{-1/2}$. These L-functions do not have an Euler product. We will talk about joint work with Sandro Bettin showing that for x chosen at random with denominator $\leq Q$, D(1/2, x)does tend to distribute according to a complex Gaussian of variance $c \log Q(\log \log Q)^3$ as Qtends to infinity. We will also discuss applications related to modular symbols. This involves tools from smooth dynamics.

SPEAKER: Nirvana Coppola (VU Amsterdam)

TITLE: Reduction type of genus 3 curves in special strata of their moduli space

ABSTRACT: The goal of this talk is to give a classification of the reduction of genus-3 curves whose automorphism group contains a subgroup isomorphic to the Klein group. We will first compute all possible reduction types, and then characterize when each case is attained in terms of invariants attached to the equation of a normalized model of the curve. This is joint work with I. Bouw, P. Kılıçer, S. Kunzweiler, E. Lorenzo García and A. Somoza Henares.

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SPEAKER: Paul Ziegler (Technical University of Munich)

TITLE: Geometric stabilization via p-adic integration

ABSTRACT: The fundamental lemma is an identity of integrals playing an important role in the Langlands program. This identity was reformulated into a statement about the cohomology of moduli spaces of Higgs bundles, called the geometric stabilization theorem, and proved in this form by Ngô. I will give an introduction to these results and explain a new proof of the geometric stabilization theorem, which is joint work with Michael Groechenig and Dimitri Wyss, using the technique of *p*-adic integration.

SPEAKER: Asbjørn Nordentoft (Bonn)

TITLE: Wide moments of automorphic L-functions

ABSTRACT: Calculating the moments of L-function is a central theme in analytic number theory with applications to subconvexity and non-vanishing (which in turn has deep arithmetic implications to equidistribution and point counting problems). In this talk we will give a gentle introduction to a certain type of "wide moments", which in many cases can be calculated using geometric methods.

In particular we will consider the case of Rankin–Selberg L-functions of GL_2 automorphic forms twisted by class group characters of an imaginary quadratic field, in which case the "wide moments" are connected to equidistribution of Heegner points using Waldspurger's formula. We will also present applications to non-vanishing.

SPEAKER: Julia Brandes (Chalmers and University of Gothenburg)

TITLE: Two-dimensional Weyl sums failing square-root cancellation along lines

ABSTRACT: We show that a certain two-dimensional family of Weyl sums of length P takes values as large as $P^{3/4+o(1)}$ on almost all linear slices of the unit torus, contradicting a widely held expectation that Weyl sums should exhibit square-root cancellation on generic subvarieties of the unit torus. This is joint work with I. Shparlinski, and extends joint work with S. T. Parsell, C. Poulias, G. Shakan and R. C. Vaughan.

SPEAKER: Pavel Kurasov (Stockholm University)

TITLE: Crystalline measures: quantum graphs, stable polynomials and explicit examples ABSTRACT: Quantum graphs – differential operators on metric graphs – via trace formula lead to crystalline measures recently studied by Y. Meyer, A. Olevskii and N. Lev. Crystalline measures are tempered distributions given by locally finite purely atomic measures whose Fourier transform is also a purely atomic measure. It remained unclear whether Dirac combs provide the only type of examples of crystalline measures with uniformly discrete support. To show that the measures generated by quantum graphs are not given by Dirac combs one has to prove that the eigenvalues are linearly independent with respect to rational numbers. It appears that Lang's conjecture establishes this fact in the case of rationally independent edge lengths in the metric graph, hence one obtains the first non-trivial example of a positive crystalline measure.

We are going to show how to construct a wide family of crystalline measures using stable polynomials. The measures we obtain are:

• positive crystalline measures with uniformly discrete support;

- Fourier quasicrystals for which every arithmetic progression meets the support in a finite set;
- Fourier quasicrystals for which the support is a Delone set, but the support of the Fourier transform not.

This is a joint work with Peter Sarnak.