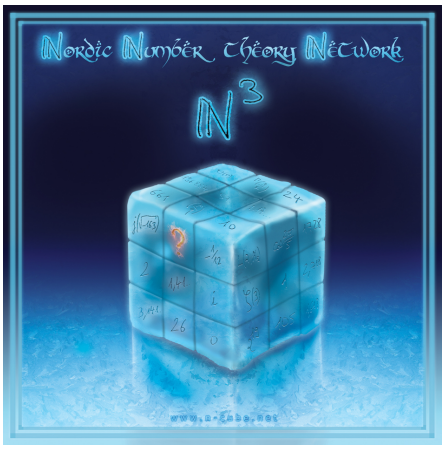
UNIVERSITY OF COPENHAGEN

DTU Danmarks Tekniske Universitet

Nordic Number theory Network Days XII

DTU Copenhagen,
May 28 – May 29, 2020
organised by Peter Beelen (DTU) and Fabien Pazuki (KU).



Program

	Thursday 28.05	Friday 29.05
13:00-13:10	*Foreword*	
13:10-14:00	**Alp Bassa**	**Maria Montanucci**
14:00-14:20	*Coffee break*	*Coffee break*
14:20-15:10	**Régis de la Bretèche**	**Baptiste Morin**
15:10-15:30	*Coffee break*	*Coffee break*
15:30-16:20	**Valentijn Karemaker**	**Myrto Mavraki**
16:20-16:40	*Coffee break*	*Farewell coffee*
16:40-17:30	**Jasmin Matz**	
17:40	*Cultural Drinks*	

Abstracts

TIME: Thursday 28, 13:10-14:00.
ROOM: Zoom.
SPEAKER: **Alp Bassa** (Bogazici University Istanbul).
TITLE: *Rational points on curves over finite fields and their asymptotic.*
ABSTRACT: Curves over finite fields with many rational points have been of interest for both theoretical reasons and for applications. To obtain such curves with large genus various methods have been employed in the past. One such method is by means of explicit recursive equations and will be the emphasis of this talk. The recursive nature of these towers makes them very special and in fact all good examples have been shown to have a modular interpretation of some sort. In this talk I will try to give an overview of the landscape of explicit recursive towers and their modularity.

TIME: Thursday 28, 14:20-15:10.
ROOM: Zoom.
SPEAKER: **Régis de la Bretèche** (University of Paris).
TITLE: *Large and small Gál sums and applications.*
ABSTRACT: We will introduce Gál sums and explain how to apply large and small values in some number theory problems.

TIME: Thursday 28, 15:30-16:20.
ROOM: Zoom.
SPEAKER: **Valentijn Karemaker** (Stockholm University and Utrecht University).
TITLE: *Mass formulae for supersingular abelian threefolds.*
ABSTRACT: Using the theory of polarised flag type quotients, we determine mass formulae for all principally polarised supersingular abelian threefolds defined over an algebraically closed field k of characteristic p. We combine these results with computations of the automorphism groups to study Oort's conjecture; we prove that every generic principally polarised supersingular abelian threefold over k of characteristic $p > 2$ has automorphism group $\mathbb{Z}/2\mathbb{Z}$. This is joint work with F. Yobuko and C.-F. Yu.

TIME: Thursday 28, 16:40-17:30.
ROOM: Zoom.
SPEAKER: **Jasmin Matz** (Copenhagen University).
TITLE: *Quantum ergodicity of compact quotients of* $\mathrm{SL}(n,\mathbb{R})$ *in the level aspect.*
ABSTRACT: For a compact Riemannian manifold M and an orthonormal basis B of $L^2(M)$ consisting of Laplace eigenfunctions, the property of M satisfying quantum ergodicity asserts that there is a sequence of density 1 of functions f in B with Laplace eigenvalue going to infinity such that the measures $|f|^2 d_M x$ weak*-converges to the Riemannian measure $d_M x$ on M. Due to work by Shnirelman and others this is known to hold for M with ergodic geodesic flow (e.g., M of negative curvature).

We want to change the perspective, considering not one fixed manifold and Laplace eigenfunctions in the high energy limit, but a sequence of Benjamini-Schramm convergent Riemannian manifolds M_j together with Laplace eigenfunctions f whose eigenvalue varies in short intervals. This perspective has been recently studied in the context of graphs by Anantharaman and Le Masson, and for hyperbolic surfaces and manifolds by Abert, Bergeron, Le Masson,

and Sahlsten. In my talk I want to discuss joint ongoing work with F. Brumley in which we study an example of higher rank, namely sequences of compact quotients of $\mathrm{SL}(n,\mathbb{R})/\mathrm{SO}(n)$.

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TIME: Friday 29, 13:10-14:00.
ROOM: Zoom.
SPEAKER: **Maria Montanucci** (DTU Copenhagen).
TITLE: *Maximal curves over finite fields.*
ABSTRACT: Algebraic curves over a finite field $\mathbb{F}_q$ and their function fields have been a source of great fascination for number theorists and geometers alike, ever since the seminal work of Hasse and Weil in the 1930s and 1940s. Many important and fruitful ideas have arisen out of this area, where number theory and algebraic geometry meet. For a long time, the study of algebraic curves and their function fields was the province of pure mathematicians. But then, in a series of three papers in the period 1977-1982, Goppa found important applications of algebraic curves over finite fields to coding theory.

The key point of Goppa's construction is that the code parameters are essentially expressed in terms of arithmetic and geometric features of the curve, such as the number N_q of $\mathbb{F}_q$-rational points and the genus g.

Goppa codes with good parameters are constructed from curves with large N_q with respect to their genus g. Given a smooth projective, algebraic curve of genus g over $\mathbb{F}_q$, an upper bound for N_q is a corollary to the celebrated Hasse-Weil Theorem,

$$N_q \leq q + 1 + 2g\sqrt{q}.$$

Curves attaining this bound are called $\mathbb{F}_q$-maximal. The Hermitian curve $\mathcal{H}$, that is, the plane projective curve with equation

$$X^{\sqrt{q}+1} + Y^{\sqrt{q}+1} + Z^{\sqrt{q}+1} = 0,$$

is a key example of an $\mathbb{F}_q$-maximal curve, as it is the unique curve, up to isomorphism, attaining the maximum possible genus $\sqrt{q}(\sqrt{q}-1)/2$ of an $\mathbb{F}_q$-maximal curve. Other important examples of maximal curves are the Suzuki and the Ree curves. It is a result commonly attributed to Serre that any curve which is $\mathbb{F}_q$-covered by an $\mathbb{F}_q$-maximal curve is still $\mathbb{F}_q$-maximal. In particular, quotient curves of $\mathbb{F}_q$-maximal curves are $\mathbb{F}_q$-maximal. Many examples of $\mathbb{F}_q$-maximal curves have been constructed as quotient curves $\mathcal{X}/G$ of the Hermitian/Ree/Suzuki curve $\mathcal{X}$ under the action of subgroups G of the full automorphism group of $\mathcal{X}$. It is a challenging problem to construct maximal curves that cannot be obtained in this way for some G.

In this presentation, we will describe our main contributions to the theory of maximal curves over finite fields. In particular, the following topics will be discussed:

1. how can we decide whether a given $\mathbb{F}_q$-maximal curve is a quotient of the Hermitian curve?

2. further examples of maximal curves that are not quotient of the Hermitian curve;

3. determination of the possible genera of $\mathbb{F}_q$-maximal curves, especially quotients of $\mathcal{H}$;

4. Weierstrass semigroups on maximal curves.

Based on joint work with: Daniele Bartoli, Peter Beelen, Massimo Giulietti, Leonardo Landi, Vincenzo Pallozzi Lavorante, Luciane Quoos, Fernando Torres, Giovanni Zini.

TIME: Friday 29, 14:20-15:10.
ROOM: Zoom.
SPEAKER: **Baptiste Morin** (University of Bordeaux).
TITLE: *Duality and class field theory for curves over p-adic fields.*
ABSTRACT: We give a preliminary definition of a new cohomology for varieties over p-adic fields, and prove a duality theorem for curves. This gives a new viewpoint on class field theory for curves, which is due to Shuji Saito. This is joint work in progress with Thomas Geisser.

TIME: Friday 29, 15:30-16:20.
ROOM: Zoom.
SPEAKER: **Myrto Mavraki** (University of Basel).
TITLE: *Elliptic surfaces, real sections and equidistribution.*
ABSTRACT: Suppose $E \to B$ is an elliptic surface defined over a number field. We study relations between the geometry and arithmetic of the sections of $E \to B$ and their specializations in the fibers E_t for algebraic parameters t in the curve B. Our questions are motivated by results in "unlikely intersections" and a conjecture of Zhang from 1998. We conjecture the existence of a norm on $E \wedge E$ induced by an Arakelov-Zhang pairing of metrized line bundles. We establish an equidistribution theorem for heights associated to adelically metrized real divisors on B. This allows us to describe the geometry of the set of parameters t in B at which m sections have linearly related specializations in the fiber E_t. This is joint work with Laura DeMarco (in preparation).