UNIVERSITY OF COPENHAGEN

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Nordic Number theory Network Days VI

University of Copenhagen,
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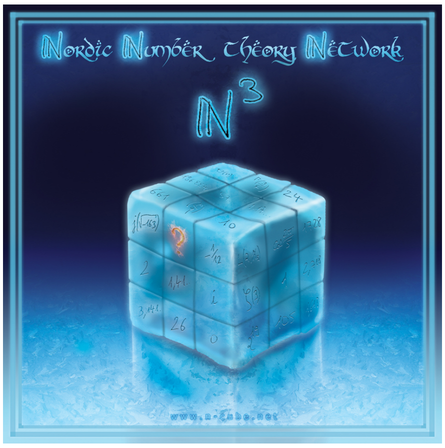

Program

	Friday 2.06	Saturday 3.06
09:10-10:00		**Lenny Taelman**
10:00-10:30		*Coffee break*
10:30-11:20		**Anna Cadoret**
11:30-12:20		**Florent Jouve**
13:00-13:05	*Foreword*	
13:05-13:55	**David Masser**	
14:05-14:55	**Wushi Goldring**	
14:55-15:20	*Coffee break*	
15:20-16:10	**Oscar Marmon**	
16:20-17:10	**Christian Liedtke**	
18:15	*Dinner*	

Abstracts

Time: Friday 2, 13:05-13:55.
Room: Aud 10.
Speaker: **David Masser** (Univ. Basel).
Title: *Unlikely intersections for algebraic curves in positive characteristic.*
Abstract: In the last two decades there has been much study of what happens when an algebraic curve in n-space is intersected with two multiplicative relations

$$x_1^{a_1} \cdots x_n^{a_n} \;=\; x_1^{b_1} \cdots x_n^{b_n} \;=\; 1 \qquad (\times)$$

for $(a_1, \ldots, a_n), (b_1, \ldots, b_n)$ linearly independent in $\mathbf{Z}^n$. Usually the intersection with the union of all $(\times)$ is at most finite, at least in zero characteristic. This often becomes false in positive characteristic, and I gave in 2014 a substitute conjecture and proved it for $n = 3$. I will describe all this together with more recent work with Dale Brownawell where we do the same for additive relations $(+)$; now an extra Frobenius structure has to be added, and there are no longer any direct analogues in zero characteristic.

Time: Friday 2, 14:05-14:55.
Room: Aud 10.
Speaker: **Wushi Goldring** (Univ. Stockholm).
Title: *Geometry engendered by G-Zips: Shimura varieties and beyond.*
Abstract: Moonen, Pink, Wedhorn and Ziegler initiated a theory of G-Zips, which is modeled on the de Rham cohomology of varieties in characteristic $p > 0$ "with G-structure", where G is a connected reductive $\mathbb{F}_p$-group. Building on their work, when X is a good reduction special fiber of a Hodge-type Shimura variety, it has been shown that there exists a smooth, surjective morphism ζ from X to a quotient stack $G - Zip^{\mu}$. When X is of PEL type, the fibers of this morphism recover the Ekedahl-Oort stratification defined earlier in terms of flags by Moonen. It is commonly believed that much of the geometry of X lies beyond the structure of ζ.

I will report on a project, initiated jointly with J.-S. Koskivirta and developed further in joint work with Koskivirta, B. Stroh and Y. Brunebarbe, which contests this common view in two stages: The first consists in showing that fundamental geometric properties of X are explained purely by means of ζ (and its generalizations). The second is that, while these geometric properties may appear to be special to Shimura varieties, the G-Zip viewpoint shows that they hold much more generally, for geometry engendered by G-Zips: Any scheme Z equipped with a morphism to $G - Zip^{\mu}$ satisfying some general scheme-theoretic properties. To illustrate our program concretely, I will describe results and conjectures regarding two basic geometric questions about X, Z: (i) Which automorphic vector bundles on X, Z admit global sections? (ii) Which of these bundles are ample? As a corollary, we also deduce old and new results over the complex numbers. Question (i) was inspired by a conjecture of F. Diamond on Hilbert modular forms mod p.

Time: Friday 3, 15:20-16:10.
Room: Aud 10.
Speaker: **Oscar Marmon** (Univ. Copenhagen).
Title: *Rational points on quartic hypersurfaces.*
Abstract: By work of Heath-Brown and Hooley, it is known that the Hasse principle holds

for non-singular cubic forms in at least nine variables. The situation for forms of higher degree is much less satisfactory. Browning and Heath-Brown established the Hasse principle for non-singular quartic forms in at least 41 variables, and Hanselmann subsequently showed that 40 variables suffice. In joint work with Pankaj Vishe, we are able to considerably reduce the number of variables needed in the quartic case. To obtain the improvement, we combine Heath-Brown's delta-symbol version of the circle method with a van der Corput differencing technique.

Time: Friday 3, 16:20-17:10.
Room: Aud 10.
Speaker: **Christian Liedtke** (TU Münich).
Title: *Crystalline Galois Representations arising from K3 Surfaces.*
Abstract: Let K be a p-adic field, let X be a K3 surface over K, and assume that X has potential semi-stable reduction. Then, we show that the following are equivalent:
1) The ℓ-adic Galois representation on $H^2(\bar{X}, \mathbb{Q}_\ell)$ is unramified for one ℓ different from p.
2) The ℓ-adic Galois representation on $H^2(\bar{X}, \mathbb{Q}_\ell)$ is unramified for all ℓ different from p.
3) The p-adic Galois representation on $H^2(\bar{X}, \mathbb{Q}_p)$ is crystalline.
4) The surface has good reduction after an unramified extension of K.

This is an analog of the classical Serre-Tate theorem for Abelian varieties. We also show by counter-examples that neither 1), nor 2), nor 3) implies that X has good reduction of X over K. However, in this case X admits a proper model over O_K, whose special fiber X_0 has at worst canonical singularities. Now, if the Galois-representation on $H^2(\bar{X}, \mathbb{Q}_p)$ is crystalline, then functors of Fontaine and Kisin provide us with an F-crystal over $W(k)$ that looks like the crystalline cohomology of some smooth K3 surface - in fact, we will show that this is the crystalline cohomology of the minimal resolution of singularities of X_0. In my talk, I will introduce all the above notions and functors (which will not give me much time to give proofs). Part of this is joint with Matsumoto, part of this is joint with Chiarellotto and Lazda.

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Time: Saturday 4, 9:10-10:00.
Room: Aud 10.
Speaker: **Lenny Taelman** (Univ. Amsterdam).
Title: *The equivariant Witt group.*
Abstract: Let K be the fraction field of a dvr R. Given a symmetric bilinear space V over K, and a group G acting by isometries on V we give necessary and sufficient criteria for V to contain a unimodular lattice stabilised by G. We give applications to automorphisms of complex K3 surfaces and to zeta functions of even-dimensional varieties over finite fields.

Time: Saturday 4, 10:30-11:20.
Room: Aud 10.
Speaker: **Anna Cadoret** (Ecole Polytechnique).
Title: *Families of abelian varieties with a common isogeny factor (with Akio Tamagawa).*
Abstract: I will discuss the following question, raised by Roessler and Szamuely. Let S be a a variety over a field k and A an abelian scheme over S. Assume there exists an abelian variety B over k such that for every closed point s in S, B is geometrically an isogeny factor of the fiber A_s. Then does this imply that the constant scheme $B \times k(\eta)$ is geometrically an isogeny

factor of the generic fiber A_η? When k is not the algebraic closure of a finite field, the answer is positive and follows by standard arguments from the Tate conjectures. The interesting case is when k is finite. I will explain how, in this case, the question can be reduced to the microweight conjecture of Zarhin for simple abelian varieties of type IV. This answers positively the original question when A_η has no isogeny factor of type IV. Passing by, we will also see that the assumption 'for every closed point s in S' can be weakened and that the question of Roessler and Szamuely makes sense and has possible interesting applications for ℓ-adic motives.

Time: Saturday 4, 11:30-12:20.
Room: Aud 10.
Speaker: **Florent Jouve** (Univ. Bordeaux).
Title: *Variations on Chebyshev's bias.*
Abstract: Chebyshev's bias, in its classical form, is the preponderance in "most" intervals $[2, x]$ of primes that are 3 modulo 4 over primes that are 1 modulo 4. Recently many generalizations and variations on this phenomenon have been explored. We will highlight the role played by some wide open conjectures on L-functions in the study of Chebyshev's bias. Our focus will be on two particular analogues of Chebyshev's question: the elliptic curve case and the Chebotarev variant. The talk will be based on joint work with Cha and Fiorilli.