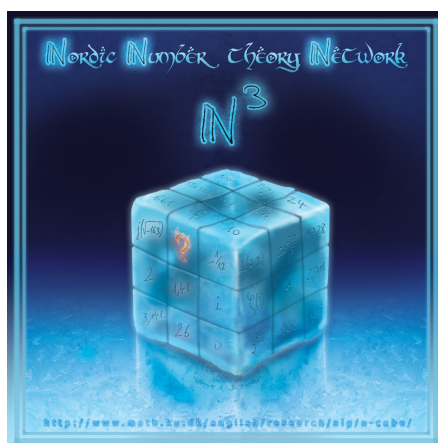

Nordic Number theory Network Days II

University of Copenhagen,
June 16 – June 17, 2015
organised by Lars Halle and Fabien Pazuki,
with the support of the Niels Bohr Professorship.



Program

	Tuesday 16.06	Wednesday 17.06
09:10-10:00		Qing Liu
10:00-10:30		<i>Coffee break</i>
10:30-11:20		Amos Turchet
11:30-12:20		Philipp Habegger
13:15-14:05	Takeshi Saito	
14:15-15:05	Michael Larsen	
15:05-15:35	<i>Coffee break</i>	
15:35-16:25	Peter Beelen	
16:35-17:25	Kalle Leppälä	
18:30	<i>Dinner</i>	

Abstracts

Time: Tuesday 16, 13:15-14:05.

Room: Aud 10.

Speaker: **Takeshi Saito** (Univ. Tokyo).

Title: *The characteristic cycle and the singular support of an étale sheaf.*

Abstract: We define the characteristic cycle of an étale sheaf on a smooth variety of arbitrary dimension in positive characteristic using the singular support, constructed by Beilinson very recently. The characteristic cycle satisfies a Milnor formula for vanishing cycles and an index formula for the Euler-Poincaré characteristic.

Time: Tuesday 16, 14:15-15:05.

Room: Aud 10.

Speaker: **Michael Larsen** (Univ. Indiana).

Title: *Diophantine Stability.*

Abstract: Let A be an abelian variety over a number field K and G a finite group. What can be said about Galois extensions L/K with group G such that $A(K) = A(L)$? I will discuss some recent results, including joint work with Bo-Hae Im and a recent preprint of Mazur and Rubin.

Time: Tuesday 16, 15:35-16:25.

Room: Aud 10.

Speaker: **Peter Beelen** (DTU Copenhagen).

Title: *Curves with many rational points: the latest on Ihara's constant.*

Abstract: Given a finite field \mathbb{F}_q , one may ask how many rational points (*i.e.* points defined over \mathbb{F}_q) an algebraic curve may have. Given a non-singular, absolutely irreducible and projective algebraic curve C , this quantity is usually denoted by $N_1(C)$. Hasse and Weil showed that $N_1(C)$ is at most $q + 1 + 2g(C)\sqrt{q}$, with $g(C)$ the genus of C . Ihara was interested in the maximal value of the quantity $A(q) = \limsup N_1(C)/g(C)$, where the limsup is taken over all families of algebraic curves as above with genus tending to infinity. The quantity $A(q)$ is now often called Ihara's constant. Drinfeld and Vladut showed that $A(q)$ is at most $\sqrt{q} - 1$, while Ihara himself showed that $A(q)$ is at least $\sqrt{q} - 1$ in case q is a square. For the nonsquare case it is still open what the exact value of $A(q)$ is. In this talk, I will discuss recent progress (joint work with A. Bassa, A. Garcia, and H. Stichtenoth) in which a good lower bound for $A(q)$ was obtained in case q is not a prime. Equations for algebraic curves will be obtained using the theory of Drinfeld modules and using these a sketch of the proof of the following fact will be given. Let $q = p^{2n+1}$, then $A(q)$ is bounded from below by the harmonic mean of $p^{n+1} - 1$ and $p^n - 1$. This lower bound is currently the best known lower bound for $A(q)$ in case $q = p^{2n+1}$ for $n > 0$.

Time: Tuesday 16, 16:35-17:25.

Room: Aud 10.

Speaker: **Kalle Leppälä** (Univ. Aarhus).

Title: *Constructive Diophantine approximation on generalized continued fraction Cantor sets.*

Abstract: Given two finite sets A and B , we ask what irrationality exponents are possible for

the numbers

$$\frac{a_1}{b_1 + \frac{a_2}{b_2 + \dots}},$$

where $a_n \in A$ and $b_n \in B$ for all n . It turns out that sometimes all exponents are possible and sometimes not. Our constructions make use of the matrix interpretations of simple continued fractions and Raney's algorithm for performing linear fractional transformations.

Time: Wednesday 17, 9:10-10:00.

Room: Aud 10.

Speaker: **Qing Liu** (Univ. Bordeaux).

Title: *Very strong approximation for certain algebraic varieties.*

Abstract: Let F be a global function field. We show that the Brauer-Manin condition on adelic points for subvarieties of a torus T over F cuts out exactly the rational points. As an application, we prove a conjecture of Harari-Voloch over global function fields which states, roughly speaking, that on any rational hyperbolic curve, the local integral points with the Brauer-Manin condition are the global integral points.

Time: Wednesday 17, 10:30-11:20.

Room: Aud 10.

Speaker: **Amos Turchet** (Chalmers Univ., Göteborg).

Title: *Uniformity results in Diophantine Geometry.*

Abstract: In 1997 Caporaso, Harris and Mazur proved that Lang Conjecture (*i.e.* rational points in general type varieties are not Zariski dense) implies that the number of rational point in curves of genus > 1 are not only finite (Faltings's Theorem) but uniform; in particular their number depends only on the genus and on the base field. This result has been extended to surfaces by Hassett. The key point is a "fibered power theorem" which has been proved to hold in arbitrary dimension by Abramovich. Analogous problems have been treated for (stably) integral points for elliptic curves and principally polarised abelian varieties. In my talk I will give a survey of these results deepening the interplay of geometry and arithmetic, as well as introducing a work in progress project, joint with Kenneth Ascher, aiming to extend the results for integral points for all log general type surfaces.

Time: Wednesday 17, 11:30-12:20.

Room: Aud 10.

Speaker: **Philipp Habegger** (Univ. Basel).

Title: *Curves of Genus 2 with Bad Reduction and Complex Multiplication.*

Abstract: If a smooth projective curve of positive genus which is defined over a number field has good reduction at some finite place, than so does its jacobian variety. It is also known that the converse is false and that this happens already in genus 2. To study the extent of this failure we investigate curves whose jacobian varieties have complex multiplication. This forces the jacobians to have potentially good reduction at all finite places by a theorem of Serre and Tate. In this talk I will present and make precise the following result. There are only finitely many curves of genus 2, defined over an algebraic closure of the rationals, which have good stable reduction everywhere and whose jacobians have complex multiplication and satisfy some further (but possibly unnecessary) restrictions. This is joint work with Fabien Pazuki.