

## Number Theory Days I

University of Copenhagen,  
December 15 – December 16, 2014  
organised by Lars Halle and Fabien Pazuki,  
with the support of the Niels Bohr Professorship.

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## Program

	Monday 15.12	Tuesday 16.12
09:10-10:00		Dan Petersen
10:10-11:00		Per Salberger
11:10-12:00		Nadim Rustom
13:15-14:05	Simon Kristensen	
14:10-15:00	Tomas Persson	
15:15-16:15	Lars Winter Christensen	
16:30-17:20	Ulf Kühn	

## Abstracts

Time: Monday 15, 13:15-14:05.

Room: S11 D-building.

Speaker: **Simon Kristensen** (Univ. Aarhus).

Title: Multiplicative problems in Diophantine approximation.

Abstract: Multiplicative Diophantine approximation is concerned with the simultaneous approximation of several real numbers by rationals with the same denominator. Where the classical theory requires all approximations to be very good, the multiplicative setup allows for some to be fairly poor, as long as others are very good. The most famous problem in the field is the celebrated conjecture of Littlewood, stating that for any pair  $(x, y)$ , the quantity

$$\liminf_{q \rightarrow \infty} q \|qx\| \|qy\| = 0,$$

where  $\|x\|$  denotes the distance from  $x$  to the nearest integer. This conjecture, originally set as an exercise to students, is still unsolved after 80 years.

In the talk, we will discuss how methods from uniform distribution theory and fractal geometry can be applied to problems in multiplicative Diophantine approximation. As a consequence of our approach, we obtain an extension of a result of Pollington and Velani on the Littlewood conjecture with a simpler proof.

Time: Monday 15, 14:10-15:00.

Room: S11 D-building.

Speaker: **Tomas Persson**.

Title: Hausdorff dimensions of sets in Diophantine approximation.

Abstract: I will talk about some joint work with Henry Reeve on a method to determine the Hausdorff dimensions of sets that appear in Diophantine approximation.

Time: Monday 15, 15:15-16:15.

Room: Aud 10 (joint with Alg/Top seminar).

Speaker: **Lars Winther Christensen**.

Title: Tate (co)homology over associative rings.

Abstract: Tate (co)homology was originally defined for modules over group algebras. The cohomological theory has a very satisfactory generalization, known as Tate-Vogel cohomology or stable cohomology or P-complete cohomology, to the setting of associative rings. The properties of the corresponding generalization of the homological theory are have hitherto been poorly understood; I will report on recent progress in this direction.

Time: Monday 15, 16:30-17:20.

Room: Aud 6.

Speaker: **Ulf Kühn**.

Title: On the generators of a certain algebra of multiple  $q$ -zeta values.

Abstract: The  $q$ -analogue of multiple zeta values given by the generating series of bi-multiple divisor sums, also referred to as bi-brackets, naturally contains the algebra of generating series of multiple divisor sums as well as the ring of quasi-modular forms. The bi-brackets satisfy a variation of the double shuffle relations. This allows us to study the number of generators via the linearised version of these relations. We obtain this way upper bounds for the dimension of this bi-filtered algebra for small length similar to the case of multiple zeta values.

Time: Tuesday 16, 9:10-10:00.

Room: Aud 1.

Speaker: **Dan Petersen**.

Title: Cohomology of local systems on the moduli of principally polarized abelian surfaces.

Abstract: Let  $A_2$  be the moduli space of principally polarized abelian surfaces. To any irreducible representation of  $\mathrm{Sp}(4)$  one may attach a local system on  $A_2$ . I will report on work in which I determined the  $\ell$ -adic cohomology of these local systems in arbitrary degree. This has applications to the study of vector-valued Siegel modular forms and to moduli of curves.

Time: Tuesday 16, 10:10-11:00.

Room: Aud 1.

Speaker: **Per Salberger**.

Title: Counting rational points on projective varieties.

Abstract: Let  $X$  be a subvariety of  $\mathbb{P}^n$  defined over  $\mathbb{Q}$  with infinitely many rational points

and  $N(X; B)$  be the number of rational points on  $X$  of height at most  $B$ . It is then a central problem in Diophantine geometry to study the asymptotic behaviour of  $N(X; B)$  when  $B$  goes to infinity. We describe some recent results on conjectures by Manin and Serre concerning this problem. One of the main tools is a global version of a "determinant method" developed by Bombieri-Pila and Heath-Brown..

Time: Tuesday 16, 11:10-12:00.

Room: Aud 1.

Speaker: **Nadim Rustom.**

Title: Finiteness questions in the arithmetic of modular forms mod  $p^m$ .

Abstract: Fix a level  $N$ , and a prime  $p$  not dividing  $N$ . For a Hecke eigenform  $f$ , let  $K_{f,p}$  be the field defined over  $\mathbb{Q}_p$  by the coefficients of  $f$ . Buzzard asked whether the degree of  $K_{f,p}$  is bounded independently of the weight of  $f$ . In joint work with Ian Kiming and Gabor Wiese, we show that Buzzard's question is closely related to questions of finiteness of (1) isomorphism classes of modular Galois representations modulo prime powers, and (2) congruence classes of eigenforms modulo prime powers. In this talk, we explain this connection, state partial results, and formulate precise conjectures.