Collection of abstracts

N-cube Days X and Number Theory Days

May 20-May 24, 2019

Åbo

N-CUBE DAYS X

REPRESENTATION OF INTEGERS BY BINARY FORMS

Shabnam Akhtari, *University of Oregon and Max Planck Institute, Bonn*

I will discuss old and new results about Thue equations and inequalities. The focus will be on the number of integer solutions of equations of the form $F(x, y) = m$, where F is a binary form with integer coefficients and degree at least 3, and m is an integer. I will also talk about a joint work with Manjul Bhargava, where we showed that many Thue equations have no solutions.

A-schemes and affine schemes in Arakelov geometry

François Charles, *University of Paris-Sud*
Joint work with Jean-Benoît Bost.

We will introduce the notion of A-schemes, which are schemes over the integers with an archimedean data. We may study coherent sheaves on those via geometry of numbers for infinite-dimensional lattices. As an application, we will discuss affineness in Arakelov geometry and draw consequences for rational points and approximation properties of holomorphic functions by integer polynomials.

Néron component groups and torsion 1-crystalline representations

***Cody Gunton**, University of Copenhagen*

Let p be any rational prime and let A be an abelian variety over a finite extension $K/\mathbb{Q}_p$ of the p-adic numbers. For any prime l distinct from p, the l-power torsion of the Néron component group of A has a description in Galois cohomology due to Grothendieck. Suppose now that A has semistable reduction. An $l = p$ complement of Grothendieck's formula established by Susan Marshall and Minhyong Kim under the restriction that the ramification e of $K/\mathbb{Q}_p$ satisfies $e < p - 1$. Using Kato's monodromy structures on finite flat group schemes and a result of Ozeki about torsion crystalline representations, we remove the restriction on e, giving a complete cohomological description of the Néron component group of A.

LARGE VALUES OF HARDY'S Z-FUNCTION

***Kamalakshya Mahatab**, Norwegian University of Science and Technology*

Let $Z(t) := \zeta\left(\frac{1}{2}+it\right)\chi^{-\frac{1}{2}}\left(\frac{1}{2}+it\right)$ be Hardy's function, where the Riemann zeta function $\zeta(s)$ has the functional equation $\zeta(s)=\chi(s)\zeta(1-s)$.Hardy's function has been used to compute the zeros of $\zeta\left(\frac{1}{2}+it\right)$ and plays a crucial role in the theory of the Riemann zeta function.In this talk we will compute the large values of $Z(t)$ and $-Z(t)$ using the resonance method.

THE ALGEBRAIC AREA OF LATTICE RANDOM WALKS

***Stéphane Ouvry**, CNRS u-psud*

We propose a formula for the enumeration of closed lattice random walks of length n enclosing a given algebraic area. The information is contained in the Kreft coefficients which encode, in the commensurate case, the Hofstadter secular equation for a quantum particle hopping on a lattice coupled to a perpendicular magnetic field. The algebraic area enumeration is possible because it is split in $2^{n/2-1}$ pieces, each tractable in terms of explicit combinatorial expressions.

Mahler measures and special values of L-functions

Riccardo Pengo, *University of Copenhagen*

We will explore the connections between special values of L-functions and the Mahler measure of polynomials, which started with the work of Boyd on Lehmer's problem for polynomials in multiple variables, and with its motivic interpretation due to Deninger. Time permitting, we will report on our work in progress to construct a polynomial whose Mahler measure is related to a given Dirichlet character using Beilinson's Eisenstein symbol in motivic cohomology.

Statistical behaviour of the Riemann zeta function on the critical line revisited

***Eero Saksman**, University of Helsinki*
Joint work with Christian Webb.

We consider statistics of the Riemann zeta function on short intervals on the critical line. Especially, we describe connection to Gaussian multiplicative chaos. This includes both rigorous results and conjectural ones.

Smooth rational curves on K3 surfaces

Matthias Schütt, *Leibniz Universität Hannover*

Smooth rational curves play a fundamental role for the structure of a K3 surface. I will first review their impact on the arithmetic of the surface, with emphasis on elliptic fibrations. Then I will focus on the case of low degree curves and explain some old and some new results for different polarisations.

On Padé approximations and Siegel's lemma

Louna Seppälä, *University of Oulu*

Padé-type approximations are a useful tool for deriving lower bounds for linear forms in some given numbers. The construction of Padé approximations leads to a large group of equations, whose solution vector, if not explicitly known, can be estimated by Siegel's lemma. Due to Bombieri and Vaaler's improved version of Siegel's lemma, the estimates for the coefficients of the Padé polynomials may be sharpened (leading to a sharper lower bound for the linear form) if the maximal minors of the coefficient matrix of the group of equations share a large common factor. We shall consider Padé-type approximations to the exponential function and see how to find a common factor from the maximal minors of the coefficient matrix of the group of equations derived from Padé approximation equations.

SPECTRAL CONSTRUCTION OF NON-HOLOMORPHIC EISENSTEIN-TYPE SERIES AND APPLICATIONS

Lejla Smajlović, *University of Sarajevo*
Joint work with Jim Cogdell and Jay Jorgenson.

We discuss construction of a holomorphic form with a divisor D on a smooth, compact, projective Kähler variety X through the application of the Kronecker limit formula to a suitable integral (over D) of an Eisenstein-type series which is defined through a series of transformations of the heat kernel K_X, associated to the Laplacian that acts on the space of smooth functions on X. The special case of X being the n-dimensional complex projective space is further discussed and application of the Kronecker limit formula to computation of Mähler measure is presented.

Number Theory Days

On periodic Dirichlet series in the extended Selberg class

Anne-Maria Ernvall-Hytönen, *Åbo Akademi University*
Joint work with Almasa Odžak and Lejla Smajlović

The structure of the extended Selberg class of degree one was completely revealed by Kaczorowski and Perelli. Recently, we gave a new characterization of the functions with periodic coefficients in that class by giving a simple relation that the coefficients have to satisfy. This criterium is particularly useful for computations and for constructing functions with specific properties. During this talk, I will briefly introduce the extended Selberg class, then give the criterium and give an idea of the proof, and finally, I will explain some applications.

On the Dirichlet series of arithmetic derivative

Pentti Haukkanen, *University of Tampere*

The arithmetic derivative D is the arithmetic function satisfying the Leibniz rule $D(mn) = D(m)n+mD(n)$ for all positive integers m, n and $D(p) = 1$ for all primes p. We show that the abscissa of convergence of the Dirichlet series of D is equal to 2, and we present this Dirichlet series in terms of the Riemann zeta-function. The same is carried out for the arithmetic partial derivative D_p and the arithmetic subderivative D_S.

Exponential sums involving Fourier coefficients of higher rank automorphic forms

Jesse Jääsaari, *University of Turku*

In this talk I will describe various conjectures concerning correlations between Fourier coefficients of higher rank automorphic forms and different exponential phases. I will also discuss recent work (partly in progress) towards some of these conjectures.

Planar additive bases: Taking additive combinatorics to a new dimension

***Jukka Kohonen**, University of Helsinki*

We present a new two-dimensional analogue of additive bases. A classical one-dimensional additive basis is a set of integers whose pairwise sums cover a given interval of integers. By analogue, a planar additive basis is a set of integer-valued points (x, y) whose pairwise sums $(x, y)+(x', y')$ cover a given rectangle of grid points in the plane. This is a natural generalization of the classical problem, and has an application in signal processing, for example in radar element placing. Somewhat surprisingly, prior research in planar bases seems almost nil. We present some exact results, some lower bounds, and some upper bounds. Reference: J. Kohonen, R. Rajamäki and V. Koivunen: Planar Additive Bases for Rectangles. Journal of Integer Sequences, Vol. 21 (2018), Article 18.9.8.

ATTACHING VALUES TO $\sum_{n=0}^{\infty} n!(\pm 1)^n$

***Tapani Matala-aho**, University of Oulu*
Joint work with Anne-Maria Ernvall-Hytönen, Louna Seppälä and Wadim Zudilin

We will present recent local-global results for the Euler's divergent series $\sum_{n=0}^{\infty} n! z^n$. It is known that $\omega_p := \sum_{n=0}^{\infty} n! s^n$, $s \in \mathbb{Z} \setminus \{0\}$, converges p-adically for every prime $p \in \mathbb{P}$. But it is not known whether ω_p is irrational for any p. What we can say is that ω_p does not satisfy any first degree B-global relation, meaning that for a fixed rational number $a/b \in \mathbb{Q}$ the relation $\omega_p = a/b$ is false in a enough big subset B of $\mathbb{P}$. For example, the set $A \cap \mathbb{P}$ with A consisting of at least $\varphi(m)/2$, $m \geq 3$, reduced residue classes $\pmod{m}$ is enough big. In the Archimedean metrics $\sum_{n=0}^{\infty} n! z^n$, $z \in \mathbb{C} \setminus \{0\}$, converges only at $z = 0$. But there are summability methods which apply and already Euler showed that $\sum_{n=0}^{\infty} n!(-1)^n = 0.5772156649\ldots$. On the other hand, attaching a meaningful value to the series $\sum_{n=0}^{\infty} n!$ has resisted all the classical summation methods.

FOURIER UNIFORMITY OF THE LIOUVILLE FUNCTION IN ALMOST ALL SHORT INTERVALS

***Kaisa Matomäki**, University of Turku*

Let λ denote the Liouville function, let $H = X^{\varepsilon}$ and write $e(z) = e^{2\pi iz}$. In this talk I will discuss a joint work with M. Radziwill and T. Tao, where we showed that, in almost all intervals of lenth H, the Liouville function does not correlate with any linear phase $e(\alpha n)$. More precisely, for almost all $x \in [X, 2X]$, one has

$$\sum_{x \leq n \leq x+H} \lambda(n) e(\alpha n) = o(H)$$

for all $\alpha \in [0, 1]$. I will also discuss some consequences of this result to correlations of arithmetic functions.

Singularity of LCM matrices on GCD closed sets with 9 elements

***Mika Mattila**, University of Tampere*

In 1976 H. J. S. Smith defined an LCM matrix as follows: let $S = \{x_1, x_2, \ldots, x_n\}$ be a set of positive integers with $x_1 < x_2 < \cdots < x_n$. The LCM matrix $[S]$ on the set S is the $n \times n$ matrix with $\mathrm{lcm}(x_i, x_j)$ as its ij entry. During the last 30 years singularity of LCM matrices has interested many authors. In 1992 Bourque and Ligh ended up conjecturing that if the GCD closedness of the set S (which means that $\gcd(x_i, x_j) \in S$ for all $i, j \in \{1, 2, \ldots, n\}$), suffices to guarantee the invertibility of the matrix $[S]$. However, a few years later this conjecture was proven false first by Haukkanen et al. and then by Hong. It turned out that the conjecture holds only on GCD closed sets with at most 7 elements but not in general for larger sets. However, the given counterexamples did not give much insight on why does the conjecture fails exactly in the case when $n = 8$. This situation was improved in two articles by Korkee et al. and Mattila et al., where a new lattice theoretic approach is introduced (the method is based on the fact that because the set S is assumed to be GCD closed, the structure $(S, |)$ actually forms a meet semilattice). In another article by Mattila et al. this lattice-theoretic method is then developed even further. Since the cases $n \leq 8$ have been thoroughly studied in the above mentioned articles, the next natural step is to apply the methods to the case $n = 9$. This was done by Altinisik and Altintas as they consider the different lattice structures of $(S, |)$ that can result as a singular LCM matrix $[S]$. However, their investigation leaves two open questions, and the main purpose of this presentation is to provide solutions to them.

Kalle Väisälä - number theorist and reformer of mathematics education

Jorma Merikoski, *University of Tampere*

Kalle Väisälä (1893-1968), a Finnish mathematician, had a short career as a number theorist and a long career as an author of text-books that reformed mathematics education in Finnish secondary schools. This topic is discussed.

A DENSITY VERSION OF WARING'S PROBLEM

Juho Salmensuu, *University of Turku*

Many additive problems have been classically treated by the Hardy-Littlewood "circle" method. In 2005 Green introduced the transference principle that shares some similarities with the circle method, but can be used to study more sparse sets than is currently possible with the circle method. We use the transference principle to study a density version of Waring's problem. We prove that if a subset of kth powers has a positive relative lower density at least $(k/(k+1))^{1/k} + \epsilon$, then all sufficiently large natural numbers can be written as the sum of k^2 elements of that set.