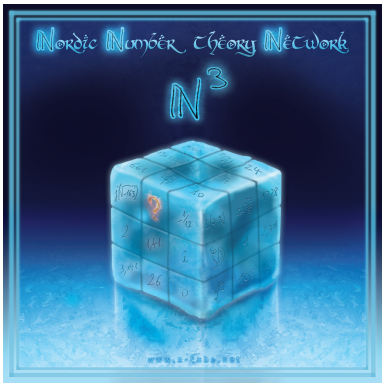
Nordic Number theory Network Days VII

Stockholm University and KTH Royal Institute of Technology
October 20–21, 2017



Schedule

Friday 20th

13:00 – 13:05	*Welcome*
13:05 – 13:55	**Rachel Newton**
14:05 – 14:55	**Morten Risager**
14:55 – 15:20	*Coffee break*
15:20 – 16:10	**Leila Schneps**
16:20 – 17:10	**Kaisa Matomäki**
18:00 –	Social Dinner

Saturday 21st

8:45 – 9:35	**Arno Kret**
9:40 – 10:30	**Ben Moonen**
10:30 – 10:55	*Coffee break*
10:55 – 11:45	**Arne Smeets**
11:50 – 12:40	**Neil Dummigan**

Location

Friday: KTH, room F3
Saturday: Stockholm University, house 5, room 14

Organizers: Jonas Bergström & Wushi Goldring (SU), Pär Kurlberg & David Rydh (KTH)
Supported by the Swedish Research Council (VR) and Stockholm Mathematics Centre (SMC).

Abstracts

Time: Friday, Oct 20, 13:05–13:55.
Room: KTH, F3
Speaker: **Rachel Newton** (University of Reading)

Title: *Transcendental Brauer groups of products of elliptic curves*
Abstract: In 1970, Manin observed that the Brauer group $\mathrm{Br}(X)$ of a variety X over a number field K can obstruct the Hasse principle on X. In other words, the lack of a K-point on X despite the existence of points over every completion of K is sometimes explained by non-trivial elements in $\mathrm{Br}(X)$. The 'algebraic' part of $\mathrm{Br}(X)$ is the part which becomes trivial upon base change to an algebraic closure of K. It is generally easier to handle than the remaining 'transcendental' part and has been widely studied. Until recently, very little was known about the transcendental part of the Brauer group.

Results of Skorobogatov and Zarhin allow one to compute the transcendental Brauer group of a product of elliptic curves. Ieronymou and Skorobogatov used these results to compute the odd order torsion in the transcendental Brauer group of diagonal quartic surfaces. The first step in their approach is to relate a diagonal quartic surface to a product of elliptic curves with complex multiplication by the Gaussian integers. I will show how to extend their methods to compute transcendental Brauer groups of products of other elliptic curves with complex multiplication. Using these results, I will give examples of Kummer surfaces where there is no Brauer–Manin obstruction coming from the algebraic part of the Brauer group but a transcendental Brauer class causes a failure of weak approximation.

Time: Friday, Oct 20, 14:05–14:55.
Room: KTH, F3
Speaker: **Morten Risager** (University of Copenhagen)

Title: *Arithmetic statistics of modular symbols*
Abstract: Mazur, Rubin, and Stein have recently formulated a series of conjectures about statistical properties of modular symbols in order to understand central values of twists of elliptic curve L-functions. Two of these conjectures relate to the asymptotic growth of the first and second moments of the modular symbols. We prove these on average by using analytic properties of Eisenstein series twisted by modular symbols. Another of their conjectures predicts the Gaussian distribution of normalized modular symbols. We prove a refined version of this conjecture.

Time: Friday, Oct 20, 15:20–16:10.
Room: KTH, F3
Speaker: **Leila Schneps** (Institut de Mathématiques de Jussieu)

Title: *Multiple zeta values in genus* 0 *and* 1
Abstract: In the classical theory of multiple zeta values, the values of convergent power series generalizing the Riemann zeta function to many variables are identified with periods of the moduli spaces $M_{0,n}$ of genus zero curves with n marked points. This point of view has led to remarkable conjectures (some partially proven) identifying various objects associated with geometry on the one hand and arithmetic on the other. We will describe in particular the known and conjectured relations between Grothendieck–Teichmüller theory and the double shuffle structure of multiple zeta values, which has been studied for several years.

In the second part of the talk we will explain the much more recent extension of the above theory to the elliptic situation. The natural extension of the geometric aspects from genus zero to genus one led to the definition of an analogous extension of multiple zeta values to elliptic multiple zeta values. The standard results and conjectures from genus zero have analogous statements in genus one, but the genus one situation also has some surprising and clarifying simplifications.

Time: Friday, Oct 20, 16:20–17:10.
Room: KTH, F3
Speaker: **Kaisa Matomäki** (University of Turku)

Title: *Correlations of von Mangoldt and higher order divisor functions*
Abstract: I will discuss joint work with M. Radziwill and T. Tao on asymptotics for the sums $\sum_{n\leq X}\Lambda(n)\Lambda(n+h)$ and $\sum_{n\leq X}d_k(n)d_l(n+h)$ where Λ is the von Mangoldt function and d_k is the kth divisor function. For the first sum we show that the expected asymptotics hold for almost all $|h|\leq X^{8/33}$ and for the second sum we show that the expected asymptotics hold for almost all $|h|\leq(\log X)^{O_{k,l}(1)}$.

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Time: Saturday, Oct 21, 8:45–09:35.
Room: SU, 5:14
Speaker: **Arno Kret** (University of Amsterdam)

Title: *Galois representations for the general symplectic group*
Abstract: In a recent preprint with Sug Woo Shin `https://arxiv.org/abs/1609.04223` I construct Galois representations corresponding to cohomological cuspidal automorphic representations of general symplectic groups over totally real number fields under the local hypothesis that there is a Steinberg component. In this talk I will explain this result and some parts of the construction.

Time: Saturday, Oct 21, 9:40–10:30.
Room: SU, 5:14
Speaker: **Ben Moonen** (Radboud University Nijmegen)

Title: *Half-twists and some cases of the Tate conjecture*
Abstract: I will explain what half-twists (a Hodge-theoretic construction due to van Geemen) are, and how this construction can be phrased in a way that is meaningful in a motivic setting. After discussing some general results about families of motives, I will then explain how, using half-twists, one can sometimes relate algebraic surfaces to abelian varieties, and prove cases of the Tate conjecture. This may be viewed as an analogue of the Kuga–Satake construction, but is actually much easier to work with.

Time: Saturday, Oct 21, 10:55–11:45.
Room: SU, 5:14
Speaker: **Arne Smeets** (Radboud University Nijmegen)

Title: *Pseudo-split fibres and a geometric Ax–Kochen theorem*
Abstract: The celebrated Ax–Kochen theorem states that for every positive integer d, there is a finite set S_d of primes such that if p is a prime not in S_d, then every homogeneous polynomial of degree d over $\mathbf{Q}_p$ in more than d^2 variables has a non-trivial zero. This classical result was originally proven using model theory.

I will present a geometric statement which generalizes the Ax–Kochen theorem and which is optimal. This uses some birational and toroidal geometry.

This is joint work with Dan Loughran and Alexei Skorobogatov, building on earlier work of Colliot-Thélène and Denef.

Time: Saturday, Oct 21, 11:50–12:40.
Room: SU, 5:14
Speaker: **Neil Dummigan** (University of Sheffield)

Title: *Automorphic forms on Feit's Hermitian lattices*
Abstract: Feit showed, in 1978, that the genus of unimodular Hermitian lattices of rank 12 over the Eisenstein integers contains precisely 20 classes. Complex-valued functions on this finite set are automorphic forms for a unitary group. Using Kneser neighbours, we find a basis of Hecke eigenforms, for each of which we propose a global Arthur parameter. This is consistent with several kinds of congruences involving classical modular forms and critical L-values, and also produces some new examples of Eisenstein congruences for $U(2,2)$.

This is joint work with Sebastian Schönnenbeck.