



## Intercity Seminar on Arakelov Geometry 2018

University of Copenhagen,  
September 3 – 7, 2018  
organised by Gerard Freixas, Lars Halle, Alberto Navarro, Fabien Pazuki.

### Program

|             | Mon 3.09            | Tue 4.09            | Wed 5.09            | Thu 6.09            | Fri 7.09         |
|-------------|---------------------|---------------------|---------------------|---------------------|------------------|
| 09:00-09:50 | <b>Bost</b>         | <b>Rössler</b>      | <b>Köhler</b>       | <b>Kramer</b>       | <i>Excursion</i> |
| 09:50-10:50 | <i>Coffee break</i> | <i>Coffee break</i> | <i>Coffee break</i> | <i>Coffee break</i> | <i>to</i>        |
| 10:50-11:40 | <b>de Jong</b>      | <b>Chen</b>         | <b>Ma</b>           | <b>Gasbarri</b>     | <i>Roskilde</i>  |
| 11:40-13:30 | <i>Lunch break</i>  | <i>Lunch break</i>  | <i>Lunch break</i>  | <i>Lunch break</i>  | <i>Viking</i>    |
| 13:30-14:20 | <b>Soulé</b>        | <b>Burgos</b>       | <b>Eriksson</b>     | <b>Künnemann</b>    | <i>Museum</i>    |
| 14:20-15:20 | <i>Coffee break</i> | <i>Coffee break</i> | <i>Coffee break</i> | <i>Coffee break</i> |                  |
| 15:20-16:10 | <b>Javanpeykar</b>  | <b>Kawaguchi</b>    | <b>Yoshikawa</b>    | <b>Bismut</b>       |                  |
| 16:20-17:10 |                     | <b>Gualdi</b>       |                     |                     |                  |
| 18:30       |                     |                     |                     | <i>Dinner</i>       |                  |

## Abstracts

Time: Monday 3, 09:00-09:50.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Jean-Benoît Bost** (Univ. Paris-Sud, France).

Title: *Mod-affine varieties in Arakelov geometry.*

Abstract: This talk will discuss some counterparts of affine schemes in the framework of Arakelov geometry. Their study relies on some formalism of "quasi-coherent sheaves on arithmetic curves", which involves non-trivial estimates on the theta invariants of Euclidean lattices, and allows one to develop various foundational results in Arakelov geometry without regularity or reducedness assumptions. (Joint work with F. Charles.)

Time: Monday 3, 10:50-11:40.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Robin de Jong** (Univ. Leiden, The Netherlands).

Title: *Faltings height and Néron-Tate height of a theta divisor.*

Abstract: We prove a formula which, given a principally polarized abelian variety  $(A, \lambda)$  over the field of algebraic numbers, relates the stable Faltings height of  $A$  with the Néron-Tate height of a symmetric theta divisor on  $A$ . Our formula completes earlier results due to Bost, Hindry, Autissier and Wagener. The proof is based on the notion of a tautological model of a principally polarized abelian variety. Along the way, we express the local Néron functions for a symmetric theta divisor at non-archimedean places in terms of tautological models and the tropical Riemann theta function. We study the case of jacobians in detail and discuss some explicit examples. This is joint work with Farbod Shokrieh.

Time: Monday 3, 13:30-14:20.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Christophe Soulé** (CNRS-IHES, France).

Title: *On the slopes of the lattice of modular forms.*

Abstract: Joint work with T.Chinburg and Q.Guignard. We consider the lattice of cuspidal modular forms of weight  $k$ , equipped with the Petersson metric. We describe the behaviour of the average of the successive maxima of this lattice when  $k$  goes to infinity.

Time: Monday 3, 15:20-16:10.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Ariyan Javanpeykar** (Univ. Mainz, Germany).

Title: *Arithmetic hyperbolicity.*

Abstract: A variety is arithmetically hyperbolic if it has only finitely many "integral" points. For instance, curves of genus at least two are arithmetically hyperbolic by Faltings's theorem (formerly Mordell's conjecture). What properties do (or should) arithmetically hyperbolic varieties have? Lang's conjecture predicts for instance that such varieties have a finite automorphism group. In this talk, we will explain how one can prove that projective arithmetically hyperbolic varieties have only finitely many automorphisms.

Time: Tuesday 4, 09:00-09:50.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Damian Rössler** (Univ. Oxford, England).

Title: *Around the theorem of the hypercube.*

Abstract: (work in progress; joint with G. Kings) The “theorem” of the hypercube is a conjectural generalisation of the classical theorem of the cube. When applied to elliptic curves, the theorem of the cube can be used to construct elliptic units. We shall explain how the “theorem” of the hypercube and its Arakelov-theoretic variants, when applied to Abelian schemes, can be used to construct elements in motivic cohomology. These elements (which specialise to elliptic units in the case of relative dimension one) should coincide with specialisations of the zero part of the abelian polylogarithm.

Time: Tuesday 4, 10:50-11:40.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Huayi Chen** (IMJ-PRG Paris, France).

Title: *Géométrie d'Arakelov sur une courbe adélique.*

Abstract: Par courbe adélique on entend un corps commutatif muni d'une famille de valeurs absolues (non-nécessairement distinctes) paramétrée par un espace de mesure de telle sorte que la valeur absolue logarithmique de chaque élément non-nul du corps est une fonction intégrable. Dans cet exposé j'expliquerai comment on peut envisager une géométrie d'Arakelov sur un tel objet en faisant des liens avec des travaux plus ou moins transversaux comme géométrie arithmétique sur un corps de type fini sur  $\mathbb{Q}$  (Moriwaki), hauteur relativement à un  $M$ -corps (Gubler), méthode de  $\mathbb{R}$ -filtration (Chen), corps de Siegel (Gaudron et Rémond). Il s'agit d'un travail en collaboration avec Atsushi Moriwaki.

Time: Tuesday 4, 13:30-14:20.

Room: Auditorium 1, Frue Plads 4.

Speaker: **José Burgos Gil** (ICMAT, Spain).

Title: *A bridge between string theory and QFT.*

Abstract: In this talk I will overview QFT and string theory and will show how the hybrid topology that relates Archimedean and non Archimedean analysis can be used as a tool to understand how string theory converges to QFT in low energies. A key tool of this analysis are the singularities of the Arakelov metric when approaching the boundary of the moduli space of marked curves.

Joint work with O. Amini, S. Bloch and J. Fresán based on an insight of P. Tourkine.

Time: Tuesday 4, 15:20-16:10.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Shu Kawaguchi** (Univ. Doshisha Kyoto, Japan).

Title: *Heights and periodic points for one-parameter families of Henon maps.*

Abstract: Henon maps are a basic class of affine plane automorphisms. In this talk, I would like to discuss arithmetic properties of families of Henon maps. For families of abelian varieties, Masser and Zannier studied unlikely intersection of simultaneous torsion sections, and since then there have been many deep studies. Note that torsion points are exactly preperiodic points under the twice multiplication map of abelian varieties. The problem of unlikely intersection of simultaneous (pre)periodic sections was first studied by Baker and DeMarco for a certain family of one-variable polynomial maps, and was further studied by Ghioca, Hsia, and Tucker among others. The motivation of this talk comes from this unlikely intersection problem for families of Henon maps. On the side of Arakelov geometry, we use Yuan's equidistribution theorem. This is joint work with Liang-Chung Hsia.

Time: Tuesday 4, 16:20-17:10.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Roberto Gualdi** (Univ. Bordeaux, France).

Title: *Height of cycles in toric varieties.*

Abstract: We present in this talk some relations between suitable heights of cycles in toric varieties and the combinatorics of the defining Laurent polynomials. To do this, we associate to any Laurent polynomial  $f$  with coefficients in an adelic field two families of concave functions on a certain real vector space: the *upper functions* and the *Ronkin functions* of  $f$ . For the choice of an adelic semipositive toric metrized divisor  $\overline{D}$ , we give upper bounds for the  $\overline{D}$ -height of a complete intersection in a toric variety in terms of the upper functions of the defining Laurent polynomials. In the one-codimensional case, we prove an exact formula relating the  $\overline{D}$ -height of a hypersurface to the Ronkin function of the associated Laurent polynomial, generalizing the well-known equality for the canonical case. Our approach involves mixed integrals, Legendre-Fenchel duality and other notions from convex geometry.

Time: Wednesday 5, 09:00-09:50.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Kai Köhler** (Univ. Düsseldorf, Germany).

Title: *Torsion forms and homogeneous spaces.*

Abstract: Results about holomorphic torsion on homogeneous spaces with respect to the action of a Lie algebra shall be discussed.

Time: Wednesday 5, 10:50-11:40.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Xiaonan Ma** (IMJ-PRG Paris, France).

Title: *Bergman kernels on punctured Riemann surfaces.*

Abstract: We consider a punctured Riemann surface endowed with a Hermitian metric which equals the Poincaré metric near the punctures and a holomorphic line bundle which polarizes the metric. We show that the Bergman kernel can be localized around the singularities and its local model is the Bergman kernel of the punctured unit disc endowed with the standard Poincaré metric. One of the technical tools is a new weighted elliptic estimate near the punctures, which is uniform with respect to the tensor power. As a consequence, we obtain an optimal uniform estimate of the supremum norm of the Bergman kernel, involving a fractional growth order of the tensor power. This holds in particular for the Bergman kernel of cusp forms of high weight of non-cocompact geometrically finite Fuchsian groups of first kind without elliptic elements.

Time: Wednesday 5, 13:30-14:20.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Dennis Eriksson** (Univ. Chalmers Göteborg, Sweden).

Title: *An invariant for Calabi-Yau manifolds through analytic torsion.*

Abstract: String theorists have predicted the existence of an invariant constructed through analytic torsion for Calabi-Yau manifolds, which should, unlike the analytic torsions themselves, only depend on the complex structure. The construction should, by mirror symmetry principles, be a birational invariant and count genus 1 curves on a Calabi-Yau 'mirror'. The construction of the invariant was done by Fang-Lu-Yoshikawa in dimension 3, and in this talk, we generalize the construction to higher dimensions. A key feature is that we can control, topologically, the asymptotic behavior of the invariant along 1-parameter degenerations. This is joint work with Gerard Freixas and Christophe Mourougane.

Time: Wednesday 5, 15:20-16:10.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Ken-Ichi Yoshikawa** (Univ. Kyoto, Japan).

Title: *Enriques  $2n$ -folds and analytic torsion.*

Abstract: In this talk, a compact connected Kähler manifold of even dimension is called simple Enriques if it is not simply connected and its universal covering is either Calabi-Yau or hyperkähler. These manifolds were studied by Boissière-Nieper-Weisskirchen-Sarti and Oguiso-Schröer, independently. We introduce a holomorphic torsion invariant of simple Enriques  $2n$ -folds and report its basic properties concerning the plurisubharmonicity and the automorphy. If time allows, we will also report an explicit formula for the invariant as a (automorphic) function on the moduli space in some cases.

Time: Thursday 6, 09:00-09:50.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Jürg Kramer** (Humboldt Univ. Berlin, Germany).

Title: *Effective sup-norm bounds on average for cusp forms of even weight.*

Abstract: Let  $\Gamma \subset \mathrm{PSL}_2(\mathbb{R})$  be a Fuchsian subgroup of the first kind acting on the upper half-plane  $\mathbb{H}$ . Consider the  $d_{2k}$ -dimensional space of cusp forms  $\mathcal{S}_{2k}^\Gamma$  of weight  $2k$  for  $\Gamma$ , and let  $\{f_1, \dots, f_{d_{2k}}\}$  be an orthonormal basis of  $\mathcal{S}_{2k}^\Gamma$  with respect to the Petersson inner product. In our talk we will give effective upper and lower bounds for the supremum of the quantity  $S_{2k}^\Gamma(z) := \sum_{j=1}^{d_{2k}} |f_j(z)|^2 \mathrm{Im}(z)^{2k}$  as  $z$  ranges through  $\mathbb{H}$ .

Time: Thursday 6, 10:50-11:40.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Carlo Gasbarri** (Univ. Strasbourg, France).

Title: *Rational vs transcendental points on analytic Riemann Surfaces.*

Abstract: Let  $(X; L)$  be a polarized smooth projective variety of dimension at least two defined over  $\mathbb{Q}$ . Let  $M$  be a Riemann surface contained in  $X(\mathbb{C})$  (which may be non algebraic). Let  $U$  be an open set of  $\Delta_1$ . The number  $A(T)$  of points of logarithmic height less than or equal to  $T$  contained in the image of  $U$  is, according to a classical theorem of Bombieri–Pila, bounded by  $\exp(\epsilon T)$ . Besides the fact that explicit examples show that this estimate is optimal, We will discuss many situations where  $A(T)$  is bounded by a polynomial in  $T$ . An important class of these examples is the leaves of smooth foliations.

We will also discuss the (mysterious) interaction within the behavior of  $A(T)$  the presence in the image of  $f$  of transcendental points (called of type  $S$ ) which satisfy inequalities similar to the standard Liouville inequality: this interaction may be resumed in the following fact:

One transcendental point of type  $S$  in the image implies few rational points in the image *and* many points of type  $S$ , or equivalently, many rational points in the image imply *no* points of type  $S$  in the image.

Time: Thursday 6, 13:30-14:20.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Klaus Künnemann** (Univ. Regensburg, Germany).

Title: *Continuity of plurisubharmonic envelopes in non-archimedean geometry and test ideals.*

Abstract: We report on joint work with Walter Gubler, Philipp Jell, and Florent Martin. Let  $L$  be an ample line bundle on a smooth projective variety  $X$  over a non-archimedean field  $K$ . For a continuous metric on  $L$ , we show that the semipositive envelope is a continuous semipositive metric on  $L$  and that the non-archimedean Monge-Ampère equation has a solution under the assumption that  $X$  is a surface defined geometrically over the function field of a curve over a perfect field  $k$  of positive characteristic. The proof follows a strategy from Boucksom, Favre and Jonsson, replacing multiplier ideals by test ideals.

Time: Thursday 6, 15:20-16:10.

Room: Auditorium 1, Frue Plads 4.

Speaker: **Jean-Michel Bismut** (Univ. Paris-Sud, France).

Title: *Hypoelliptic Laplacian and the trace formula.*

Abstract: The index theorem of Atiyah-Singer for Dirac operators can be obtained by the heat equation method, that provides an interpolation between the two sides of the formula, the global side (the index), and the geometric side (the integral of 'local' characteristic classes), with time as an interpolation parameter. I will explain how to think of Selberg's trace formula on real reductive groups as the computation of an index. We evaluate semisimple orbital integrals for the heat kernel in terms of the local geometry of closed geodesics. The arch typical example is the case where  $G = SL_2(\mathbb{R})$ , for which we recover Selberg's original formula. A key instrument is the so called hypoelliptic Laplacian, which produces the desired localization on closed geodesics. The harmonic oscillator plays a key role in its construction, as well as the Dirac operator of Kostant.

**Conference dinner:**

Thursday, September 6, 2018, at 18:30 at  
*Ristorante La Rocca*  
Vendersgade 25, 1363 København K.

**Excursion:**

Friday, September 7, 2018, from 09:00, we will go to  
*Roskilde Vikings Museum.*